

# Application of a Complex Eigenmode Solver to the TESLA 1.3 GHz Structure



TECHNISCHE  
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DARMSTADT

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# Outline

- Motivation
- Computational model
  - Problem formulation
  - Planar ports of arbitrary shape  
(here: coax lines and cylindrical beam tubes)
- Numerical examples
  - 1.3 GHz structure (single cavity)  
Summary of all modes up to the 5<sup>th</sup> dipole passband
- Summary / Outlook

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# Motivation

- Linac: Cavities

- Photograph



<http://newslines.linearcollider.org>

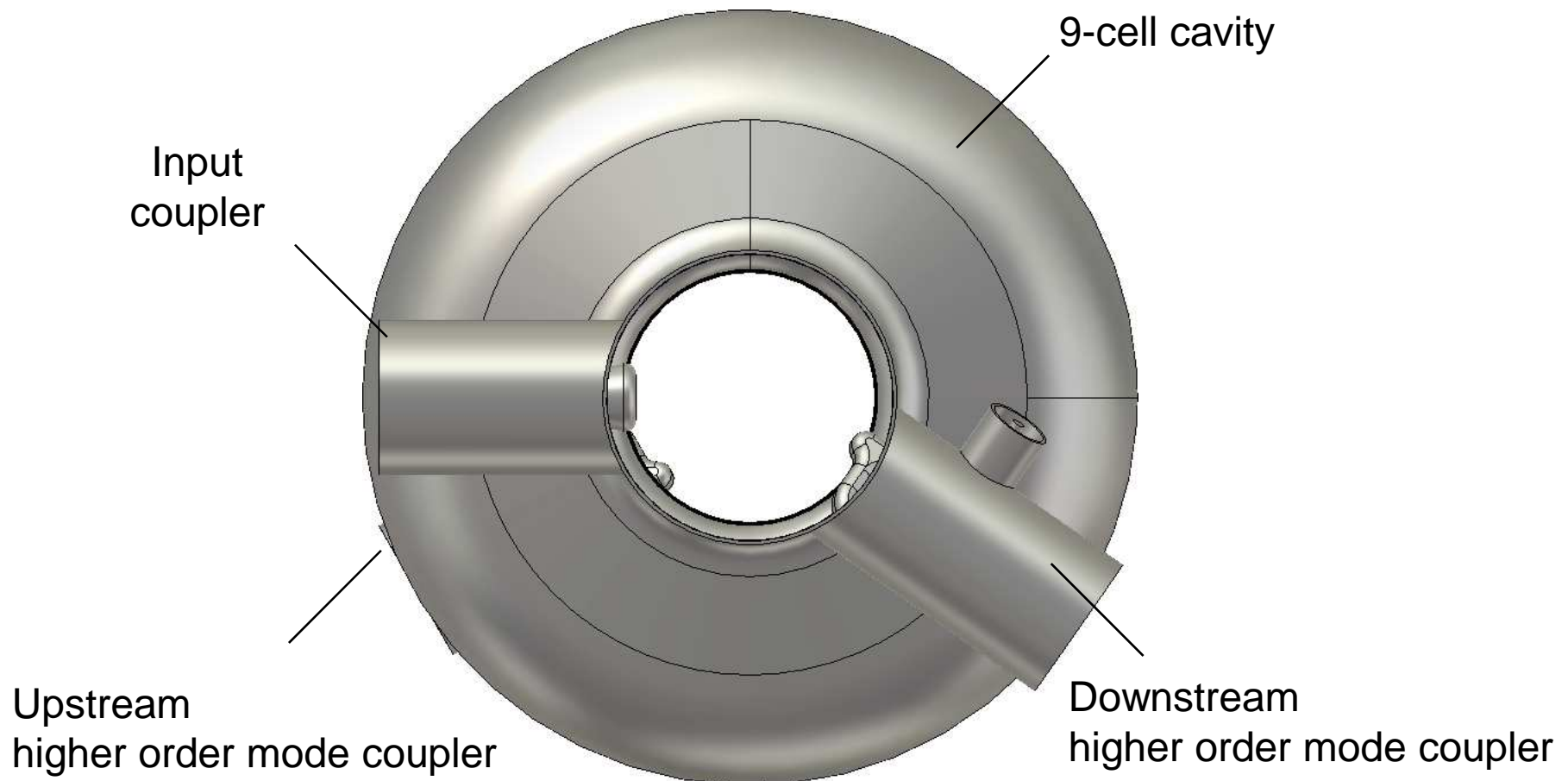
- Numerical model



CST Studio Suite 2013

# Motivation

- Superconducting resonator

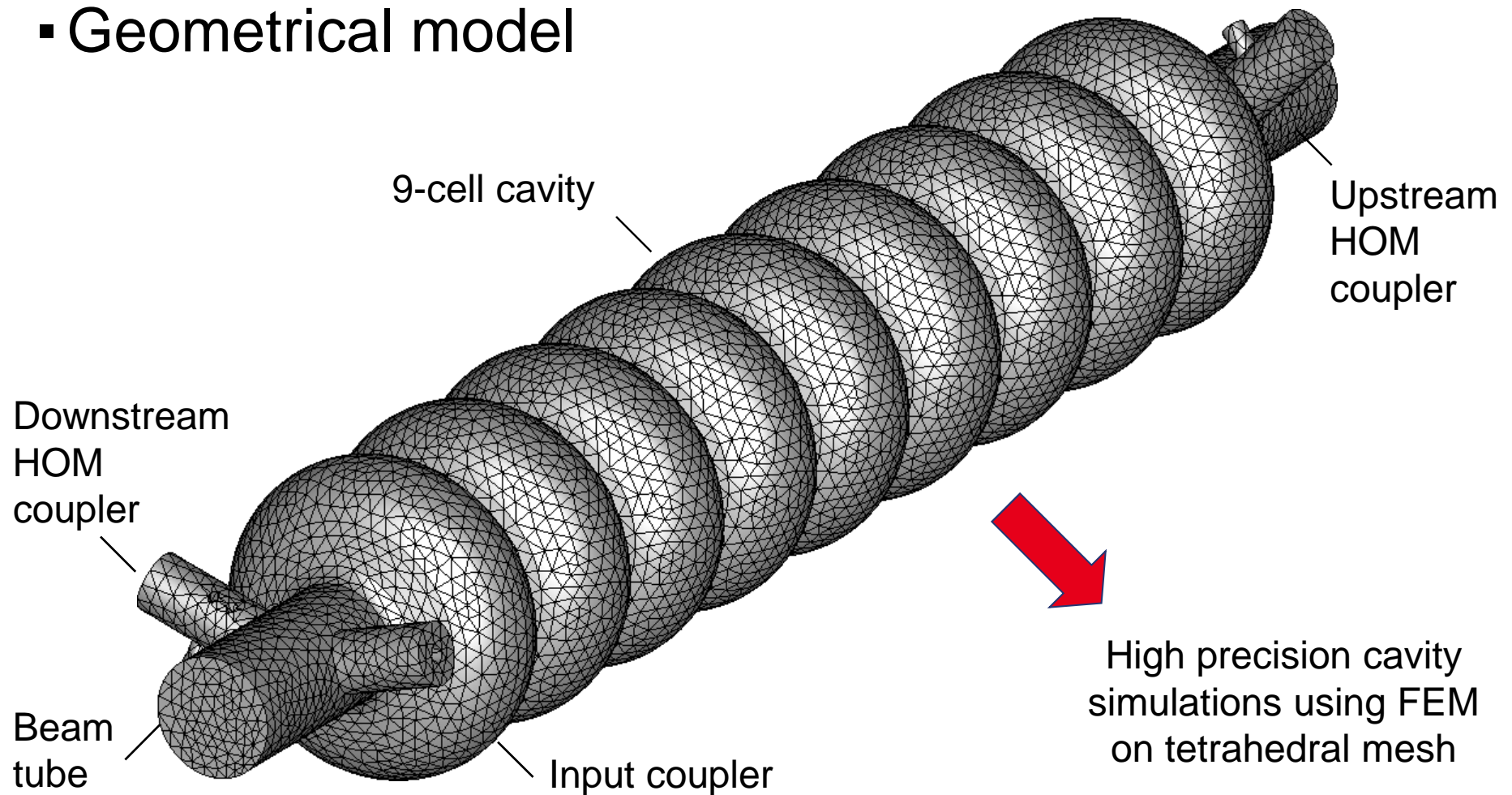


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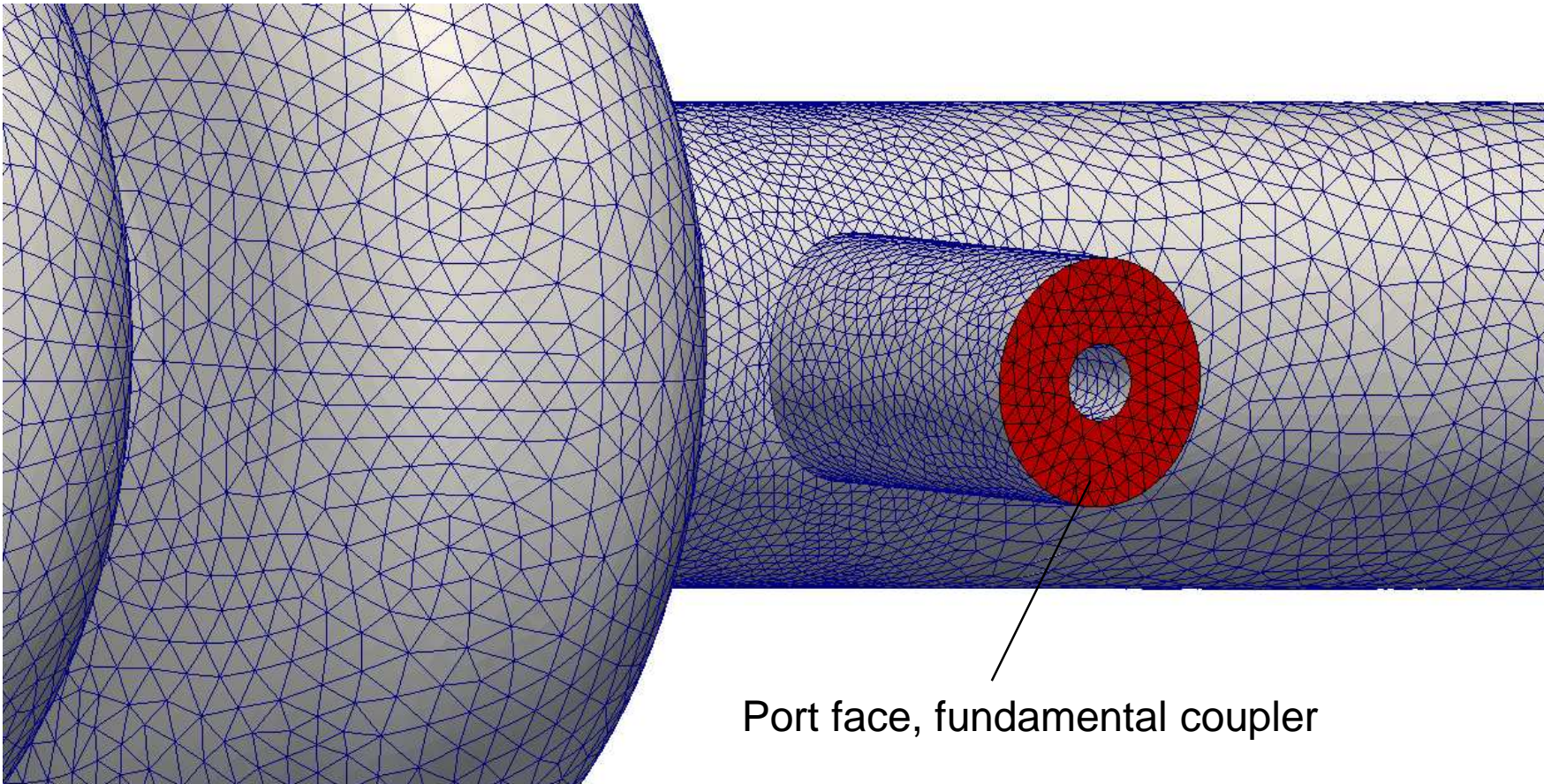
# Computational Model

- Geometrical model



# Computational Model

- Port boundary condition

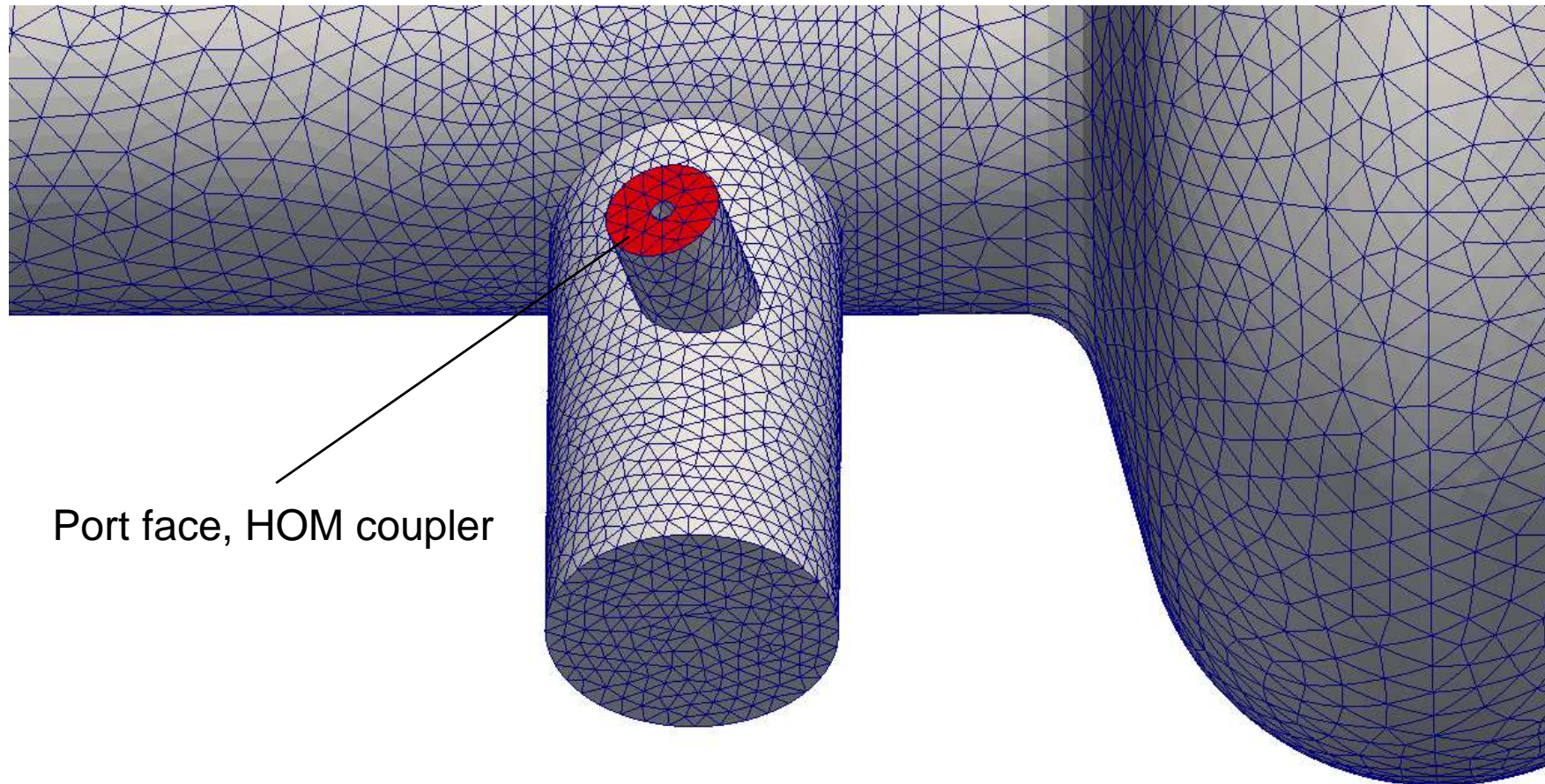


Port face, fundamental coupler



# Computational Model

- Port boundary condition

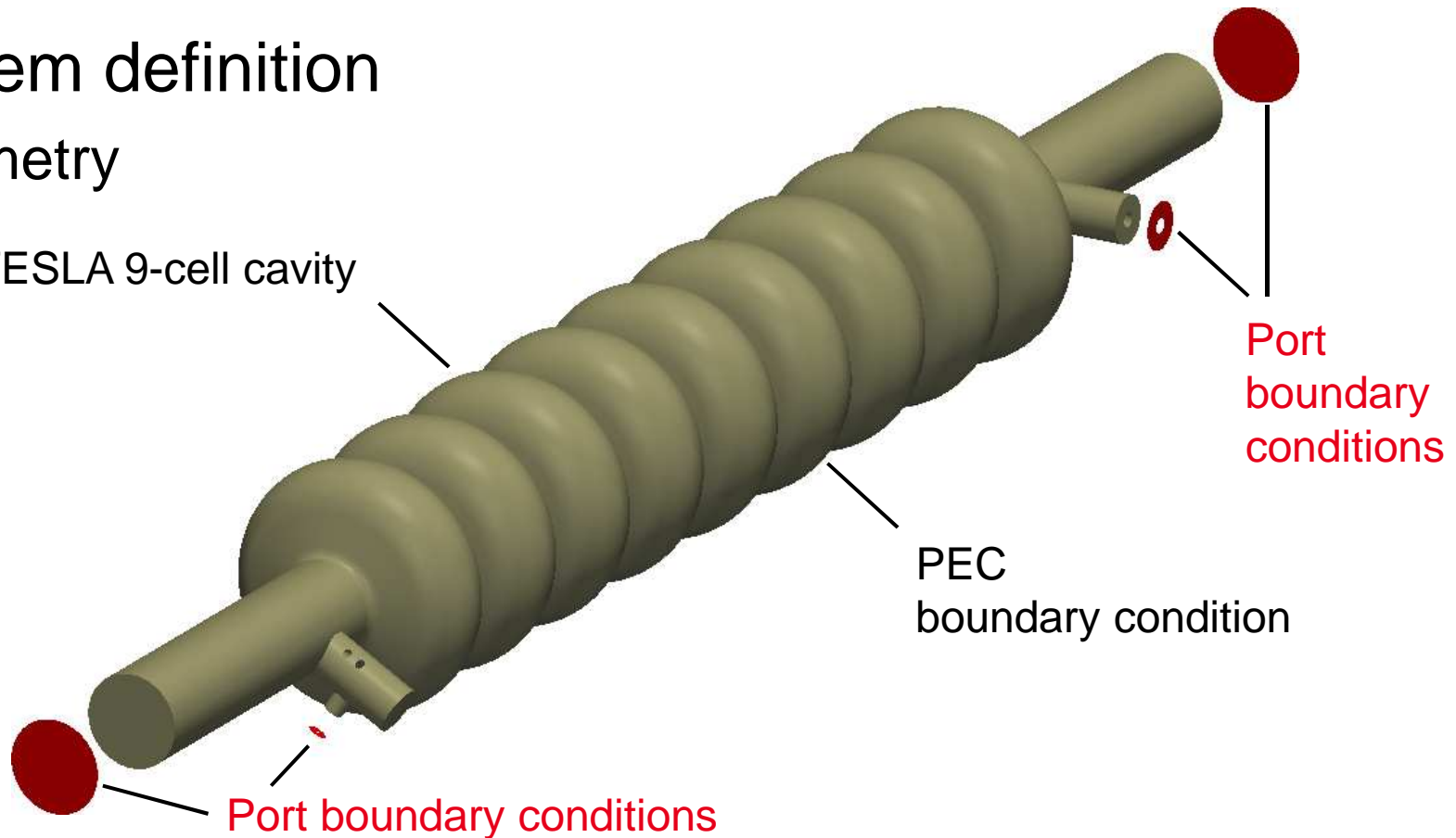


# Numerical Examples

- Problem definition

- Geometry

TESLA 9-cell cavity

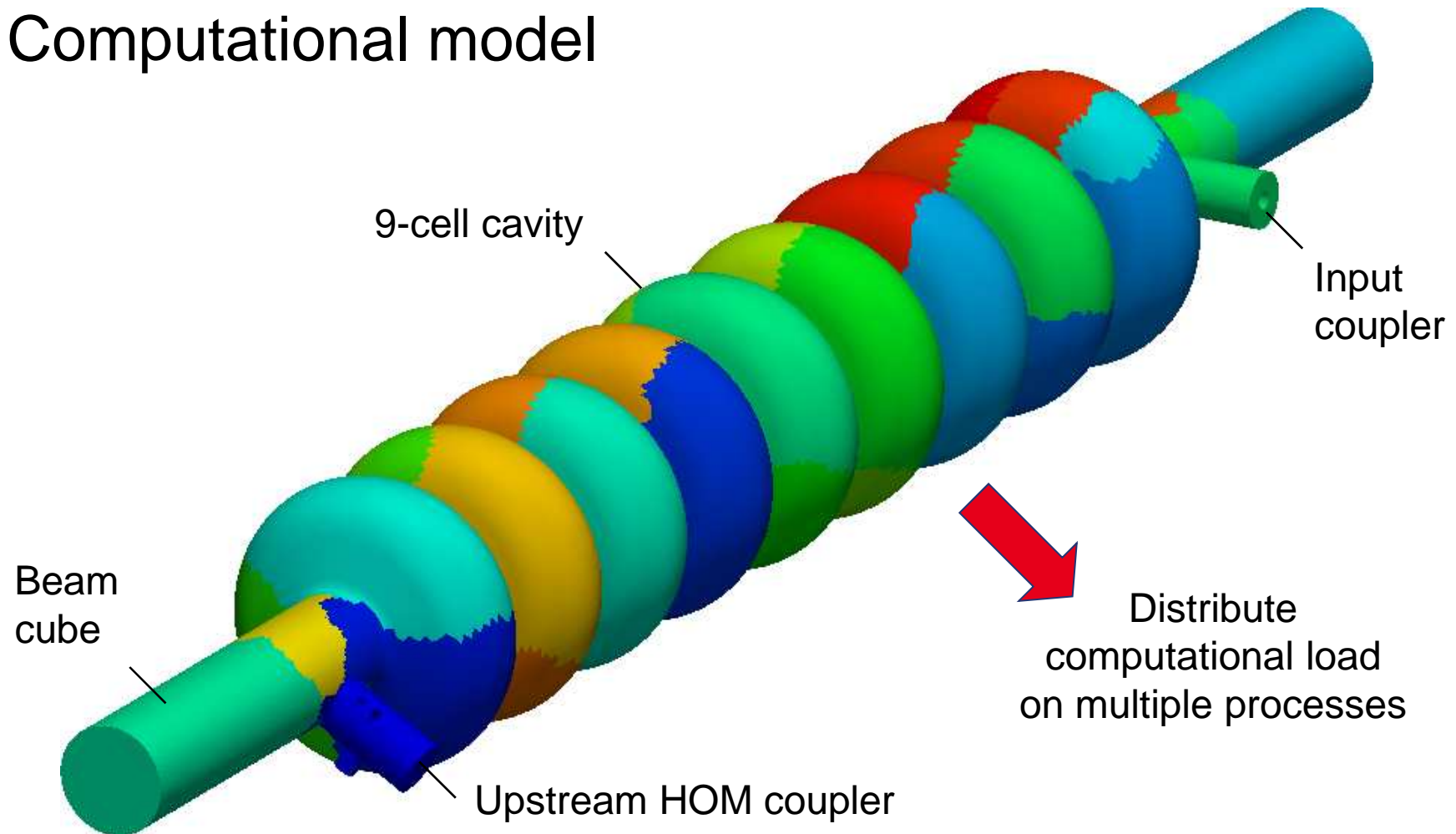


- Task

Search for the field distribution, resonance frequency and quality factor

# Numerical Examples

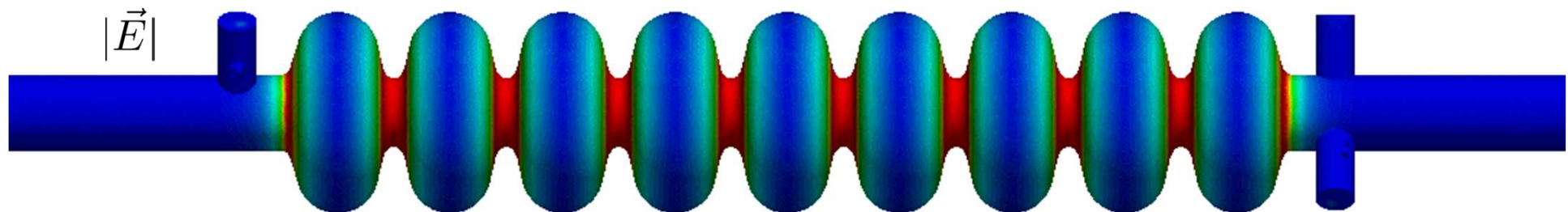
- Computational model



# Numerical Examples

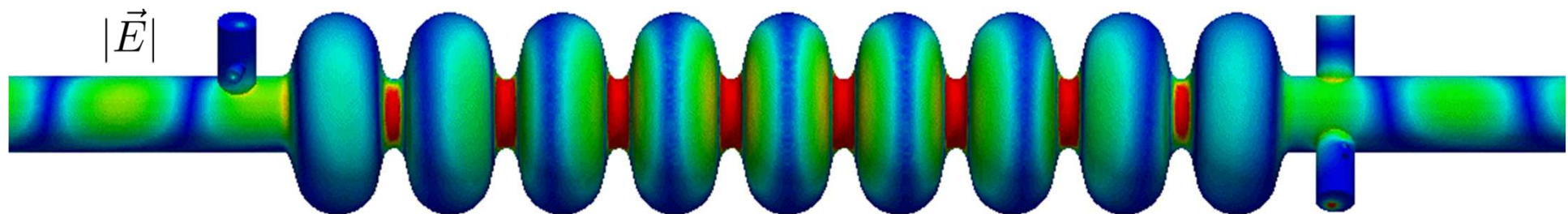
- Simulation results
  - Accelerating mode (monopole #9)

$$f_{\text{res}} = 1.300 \text{ GHz}$$
$$Q_{\text{ext}} = 2.8 \cdot 10^6$$



- Higher-order mode (dipole #37)

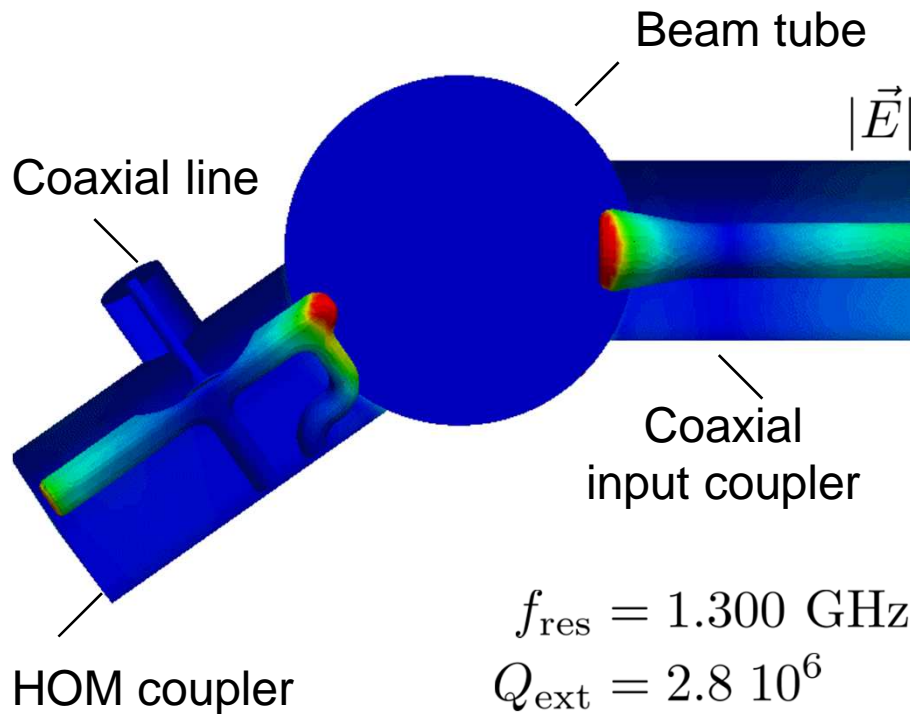
$$f_{\text{res}} = 2.476 \text{ GHz}$$
$$Q_{\text{ext}} = 1.8 \cdot 10^3$$



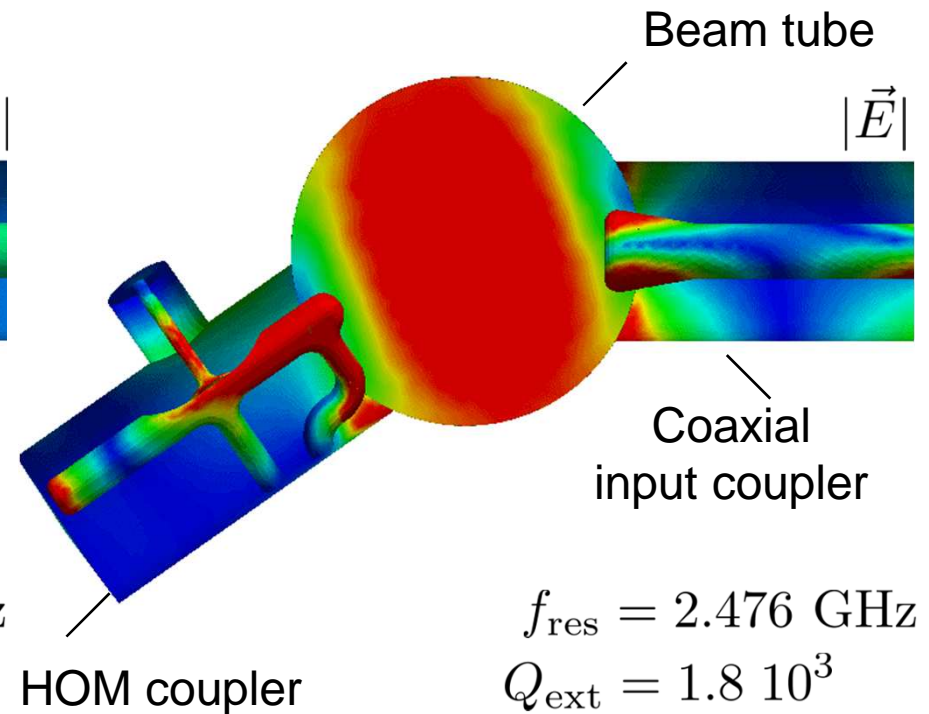
# Numerical Examples

- Simulation results

Accelerating mode  
(monopole #9)



Higher-order mode  
(dipole #37)

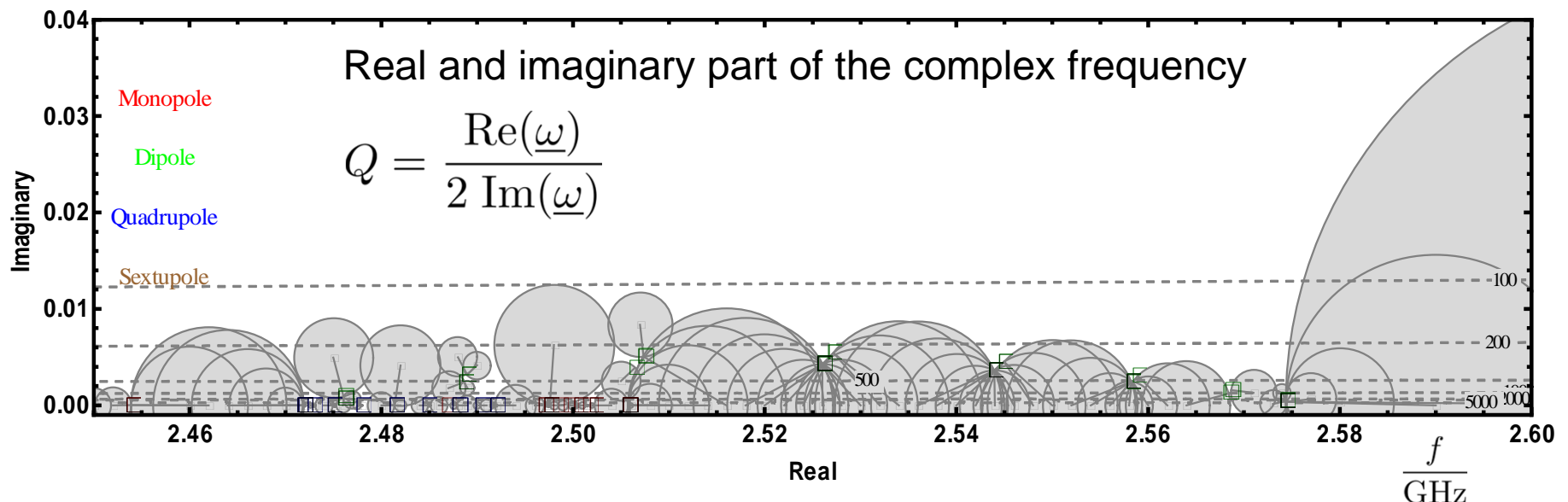


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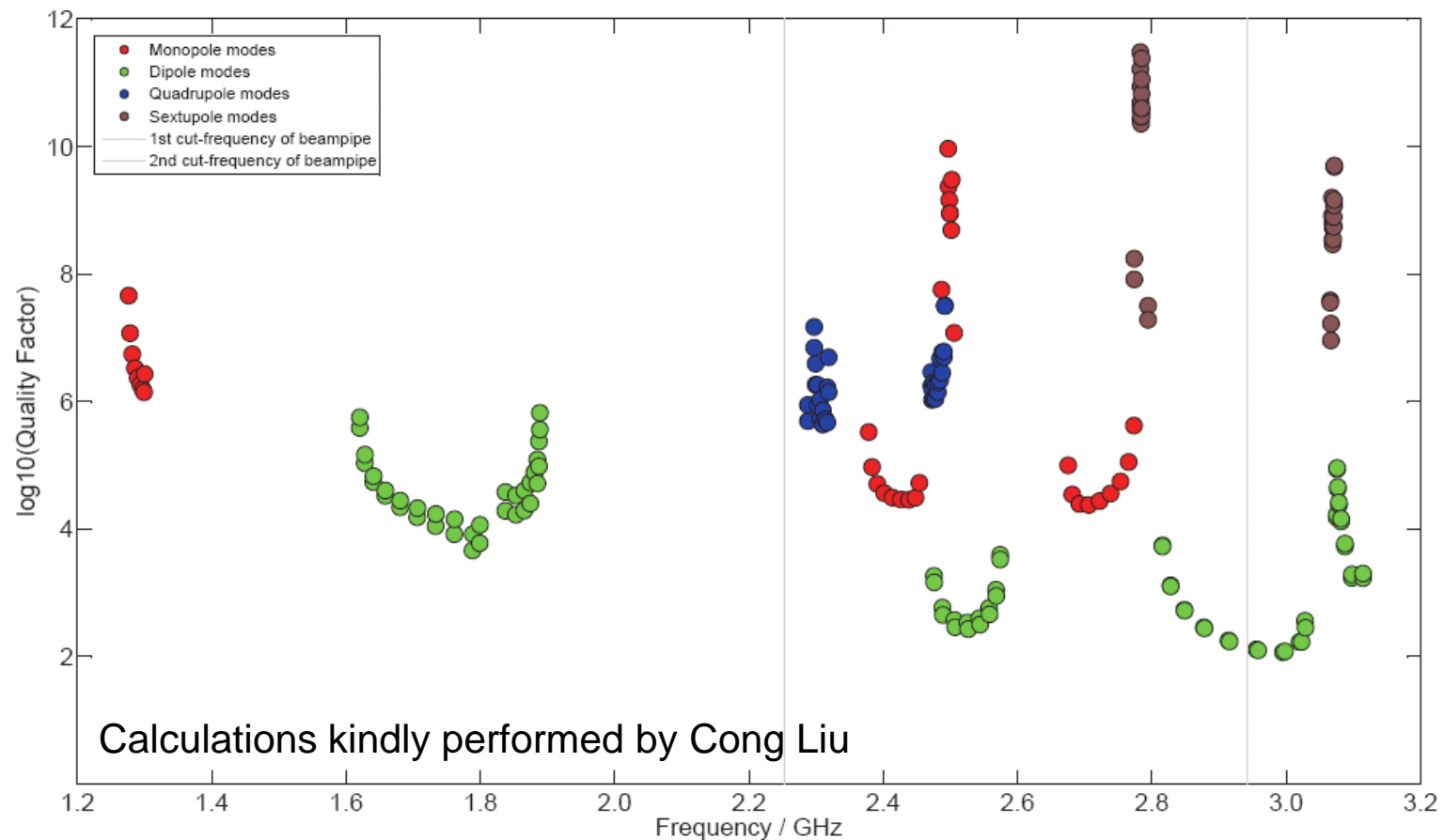
# Numerical Examples

- Controlling the Jacobi-Davidson eigenvalue solver
  - Evaluation in the complex frequency plane
  - Select best suited eigenvalues in circular region around user-specified complex target



# Numerical Examples

## Quality factor versus frequency

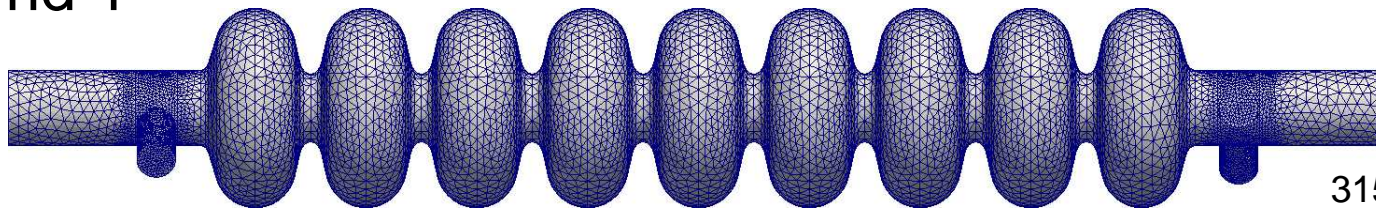




# Numerical Examples

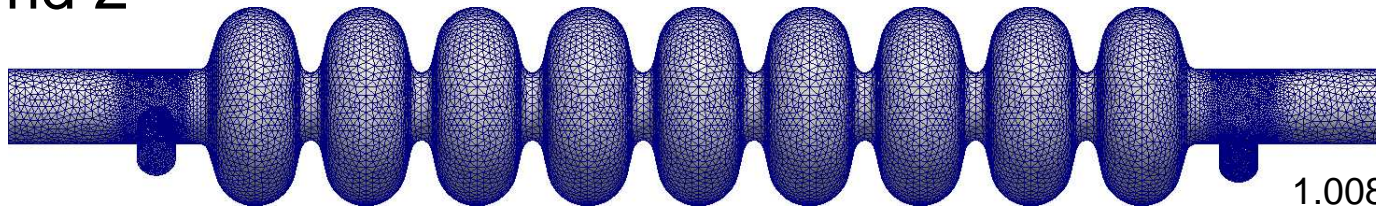
- Simulation study based on mesh density variations

- Grid 1



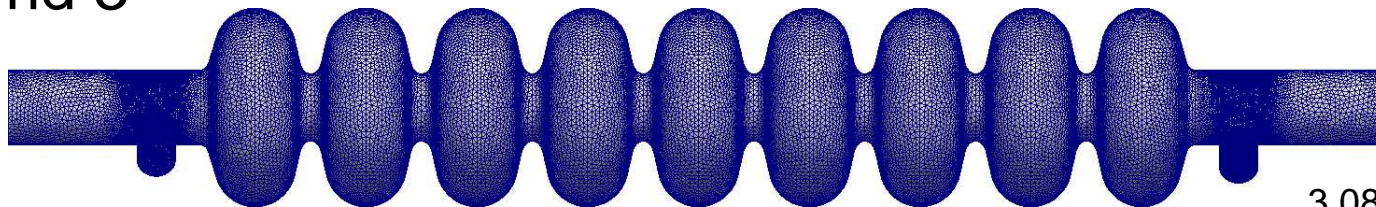
315.885 tetrahedrons  
1.932.746 complex DOF

- Grid 2



1.008.189 tetrahedrons  
6.238.328 complex DOF

- Grid 3



3.081.614 tetrahedrons  
19.177.820 complex DOF

# Numerical Examples

- Accuracy considerations (1<sup>st</sup> monopole passband)

Resonance frequency in GHz

Relative error in ppm

Mode Index	Grid Index		
	1	2	3
1	1.276304	1.276330	1.276354
2	1.278385	1.278410	1.278431
3	1.281591	1.281612	1.281628
4	1.285547	1.285565	1.285575
5	1.289785	1.289802	1.289806
6	1.293803	1.293814	1.293812
7	1.297098	1.297107	1.297099
8	1.299261	1.299267	1.299256
9	1.300011	1.300014	1.299997

Mode Index	Grid Index	
	1	2
1	-39.6	-18.8
2	-36.3	-16.4
3	-29.1	-12.4
4	-21.5	-7.9
5	-16.1	-2.7
6	-6.9	1.9
7	-1.1	5.9
8	4.0	8.4
9	10.1	13.2

Grid index:

- 1) 315.885 tetrahedrons, 1.932.746 complex DOF
- 2) 1.008.189 tetrahedrons, 6.238.328 complex DOF
- 3) 3.081.614 tetrahedrons, 19.177.820 complex DOF

$$\text{err}_\nu = \frac{f_\nu - f_3}{f_3} * 10^6$$

# Numerical Examples

- Accuracy considerations (1<sup>st</sup> monopole passband)

Quality factor in  $10^6$

Mode Index	Grid Index		
	1	2	3
1	46.323	45.444	45.151
2	11.952	11.714	11.628
3	5.581	5.475	5.433
4	3.377	3.307	3.281
5	2.372	2.324	2.305
6	1.849	1.814	1.798
7	1.568	1.537	1.522
8	1.410	1.390	1.373
9	2.791	2.679	2.645

Relative error in %

Mode Index	Grid Index	
	1	2
1	2.6	0.7
2	2.8	0.7
3	2.7	0.8
4	2.9	0.8
5	2.9	0.8
6	2.8	0.9
7	3.0	0.9
8	2.7	1.2
9	5.5	1.3

Grid index:

- 1) 315.885 tetrahedrons, 1.932.746 complex DOF
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$$\text{err}_\nu = \frac{q_\nu - q_3}{q_3} * 10^2$$

# Numerical Examples

- Accuracy considerations (1<sup>st</sup> monopole passband)

Shunt impedance in MΩ

Mode Index	Grid Index		
	1	2	3
1	0.006	0.010	0.009
2	2.078	2.020	2.006
3	0.011	0.009	0.009
4	2.069	2.003	1.981
5	0.039	0.034	0.031
6	1.989	1.947	1.928
7	0.069	0.060	0.045
8	1.974	1.914	1.901
9	1450.0	1393.0	1373.0

Relative error in %

Mode Index	Grid Index	
	1	2
1	-33.7	1.6
2	3.7	0.7
3	20.4	4.4
4	4.5	1.3
5	26.5	11.9
6	3.1	0.8
7	53.4	35.5
8	3.9	0.8
9	5.5	1.5

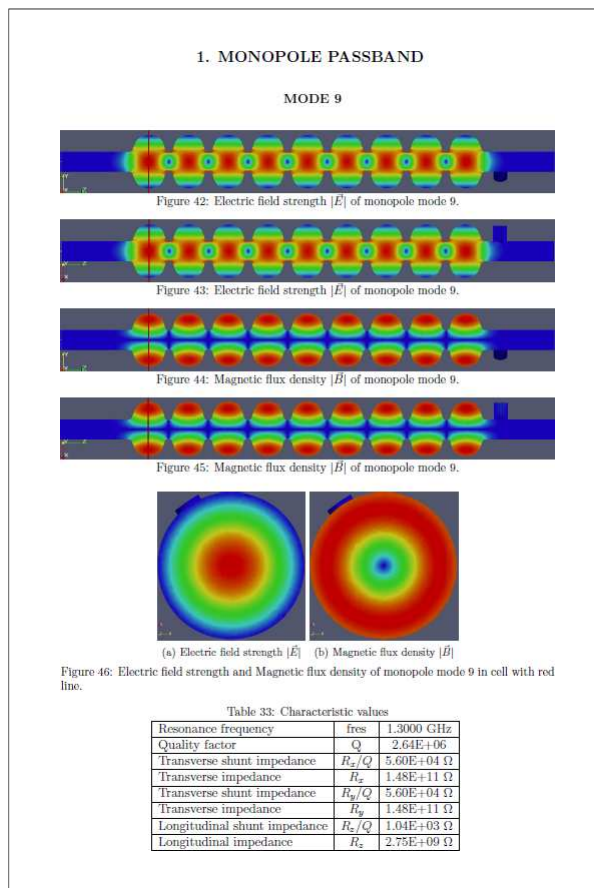
Grid index:

- 1) 315.885 tetrahedrons, 1.932.746 complex DOF
- 2) 1.008.189 tetrahedrons, 6.238.328 complex DOF
- 3) 3.081.614 tetrahedrons, 19.177.820 complex DOF

$$\text{err}_\nu = \frac{r_\nu - r_3}{r_3} * 10^2$$

# Numerical Examples

- Collection of the first 194 modes (selected page)



Magnitude of the electric field strength (longitudinal cut)

Magnitude of the magnetic flux density (longitudinal cut)

Magnitude of the electric field and the magnetic flux density (longitudinal cut)

Resonance frequency, quality factor and shunt impedances

# Numerical Examples

- Comparison to MAFIA calculations ( $f_{\text{res}}$  monopole)

	1	2	3
1	1.276	1.276	1.276
2	1.278	1.278	1.278
3	1.282	1.282	1.282
4	1.286	1.286	1.286
5	1.290	1.290	1.290
6	1.294	1.294	1.294
7	1.297	1.297	1.297
8	1.299	1.299	1.299
9	1.300	1.300	1.300
10	2.378	2.379	2.379
11	2.383	2.383	2.383
12	2.391	2.391	2.391
13	2.402	2.402	2.402
14	2.414	2.414	2.414
15	2.426	2.426	2.426
16	2.439	2.439	2.439
17	2.448	2.448	2.448
18	2.454	2.454	2.454
19	2.487	2.487	2.487
20	2.497	2.497	2.497
21	2.498	2.498	2.498
22	2.498	2.498	2.498
23	2.500	2.499	2.499
24	2.501	2.501	2.501
25	2.502	2.502	2.502
26	2.502	2.502	2.502
27	2.506	2.506	2.506
28	2.676	2.676	2.676
29	2.682	2.682	2.682
30	2.692	2.692	2.692
31	2.706	2.706	2.706
32	2.722	2.722	2.722
33	2.739	2.739	2.739
34	2.754	2.754	2.754
35	2.766	2.766	2.766
36	2.774	2.774	2.773

Mode Index

	1
1	1.276
2	1.278
3	1.281
4	1.285
5	1.289
6	1.292
7	1.296
8	1.298
9	1.298
10	2.380
11	2.386
12	2.394
13	2.406
14	2.418
15	2.431
16	2.442
17	2.450
18	2.454
19	-----
20	-----
21	-----
22	-----
23	-----
24	-----
25	-----
26	-----
27	-----
28	2.670
29	2.676
30	2.686
31	2.699
32	2.715
33	2.731
34	2.745
35	2.757
36	2.765

Mode Index

MAFIA:  
step size 1mm  
12.000 grid points

} TE modes

Reference: (TESLA 2001-33)  
Monopole, Dipole and Quadrupole  
Passbands of the TESLA 9-cell Cavity,  
R. Wanzenberg, September 14, 2001

# Numerical Examples

## Comparison to MAFIA calculations (R/Q monopole)

	1	2	3
1	0.000	0.000	0.000
2	0.174	0.172	0.173
3	0.002	0.002	0.002
4	0.613	0.606	0.604
5	0.016	0.015	0.013
6	1.076	1.073	1.072
7	0.044	0.039	0.029
8	1.400	1.377	1.385
9	519.668	519.982	520.059
10	0.313	0.309	0.310
11	0.375	0.374	0.377
12	2.459	2.459	2.456
13	1.072	1.068	1.071
14	5.879	5.890	5.904
15	3.958	3.962	3.955
16	14.575	14.561	14.603
17	84.277	84.428	84.475
18	66.772	66.968	66.931
19	$1.198 \times 10^{-7}$	$1.206 \times 10^{-7}$	$9.785 \times 10^{-8}$
20	$1.600 \times 10^{-9}$	$8.000 \times 10^{-10}$	$2.050 \times 10^{-9}$
21	$9.000 \times 10^{-10}$	$4.100 \times 10^{-9}$	$1.750 \times 10^{-9}$
22	$8.550 \times 10^{-9}$	$2.600 \times 10^{-9}$	$3.150 \times 10^{-9}$
23	$1.415 \times 10^{-8}$	$3.950 \times 10^{-9}$	$2.400 \times 10^{-9}$
24	$2.435 \times 10^{-8}$	$6.300 \times 10^{-9}$	$4.150 \times 10^{-9}$
25	$6.450 \times 10^{-9}$	$4.900 \times 10^{-9}$	$1.700 \times 10^{-9}$
26	$6.700 \times 10^{-9}$	$3.600 \times 10^{-9}$	$4.000 \times 10^{-10}$
27	$9.890 \times 10^{-8}$	$1.247 \times 10^{-7}$	$1.057 \times 10^{-7}$
28	0.060	0.064	0.066
29	0.342	0.347	0.345
30	0.008	0.009	0.009
31	0.786	0.780	0.784
32	0.075	0.072	0.072
33	0.379	0.386	0.386
34	0.137	0.140	0.139
35	0.063	0.065	0.066
36	0.027	0.028	0.028

Mode Index

	1
1	0.000
2	0.000
3	0.001
4	0.001
5	0.001
6	0.002
7	0.034
8	0.016
9	511.065
10	0.001
11	0.020
12	0.033
13	0.055
14	0.494
15	0.008
16	10.235
17	77.653
18	73.872
19	0.000
20	0.000
21	0.000
22	0.000
23	0.000
24	0.000
25	0.000
26	0.000
27	0.000
28	0.043
29	0.347
30	0.140
31	0.166
32	0.195
33	0.023
34	0.096
35	0.000
36	0.011

Mode Index

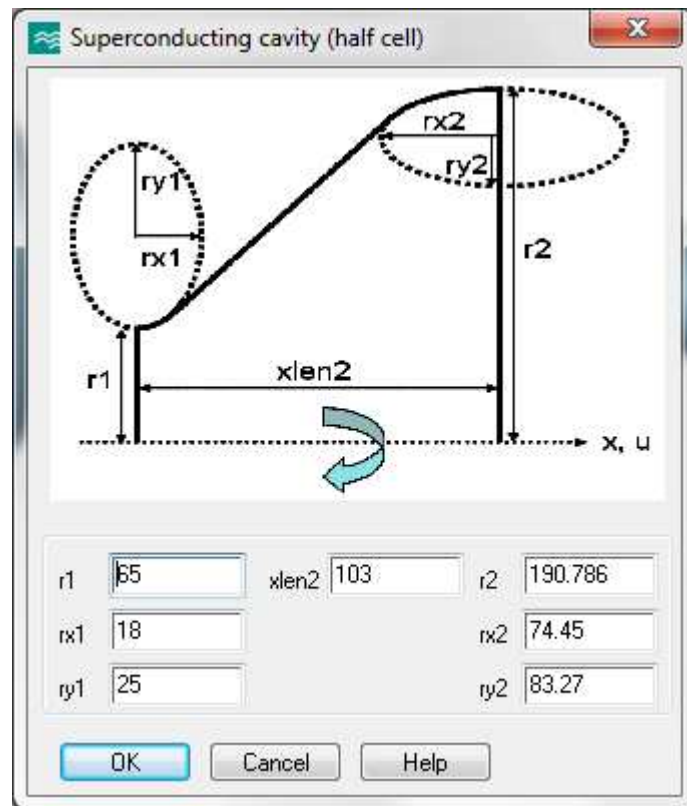
MAFIA:  
step size 1mm  
12.000 grid points

TE modes

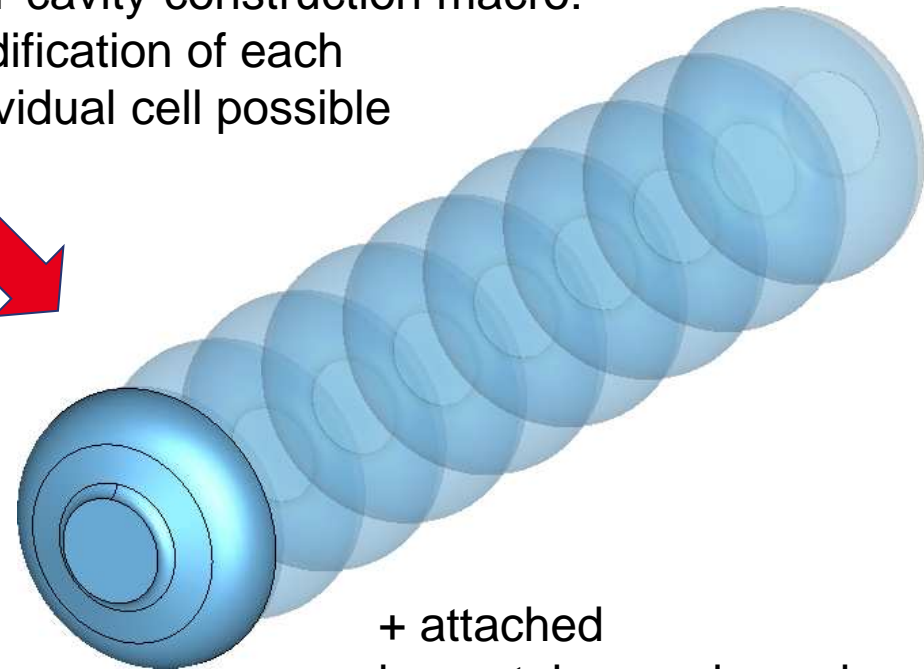
Reference: (TESLA 2001-33)  
Monopole, Dipole and Quadrupole  
Passbands of the TESLA 9-cell Cavity,  
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# Numerical Examples

- Geometry Modifications



CST cavity-construction macro:  
Modification of each  
individual cell possible

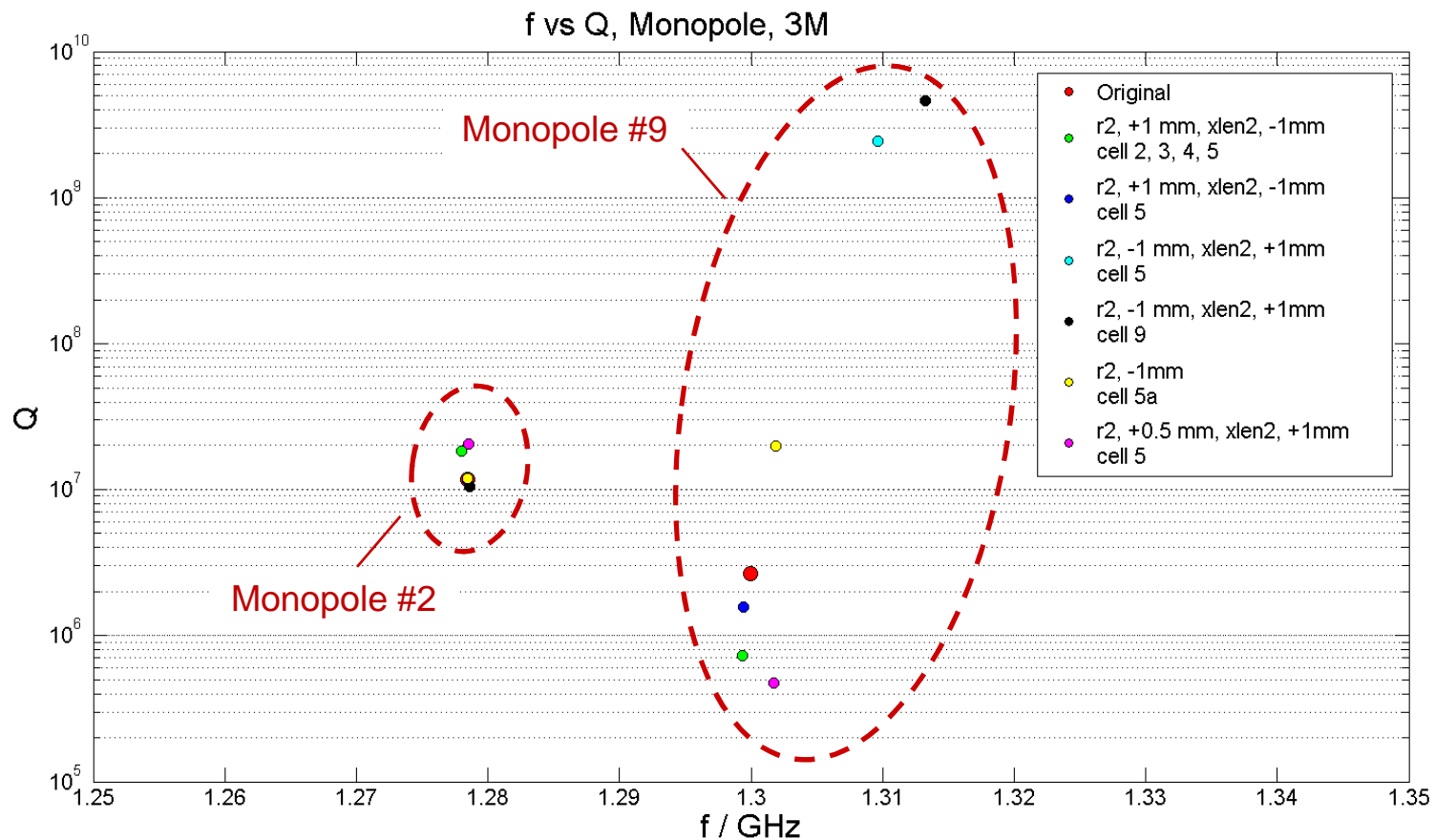


+ attached  
beam tubes and couplers



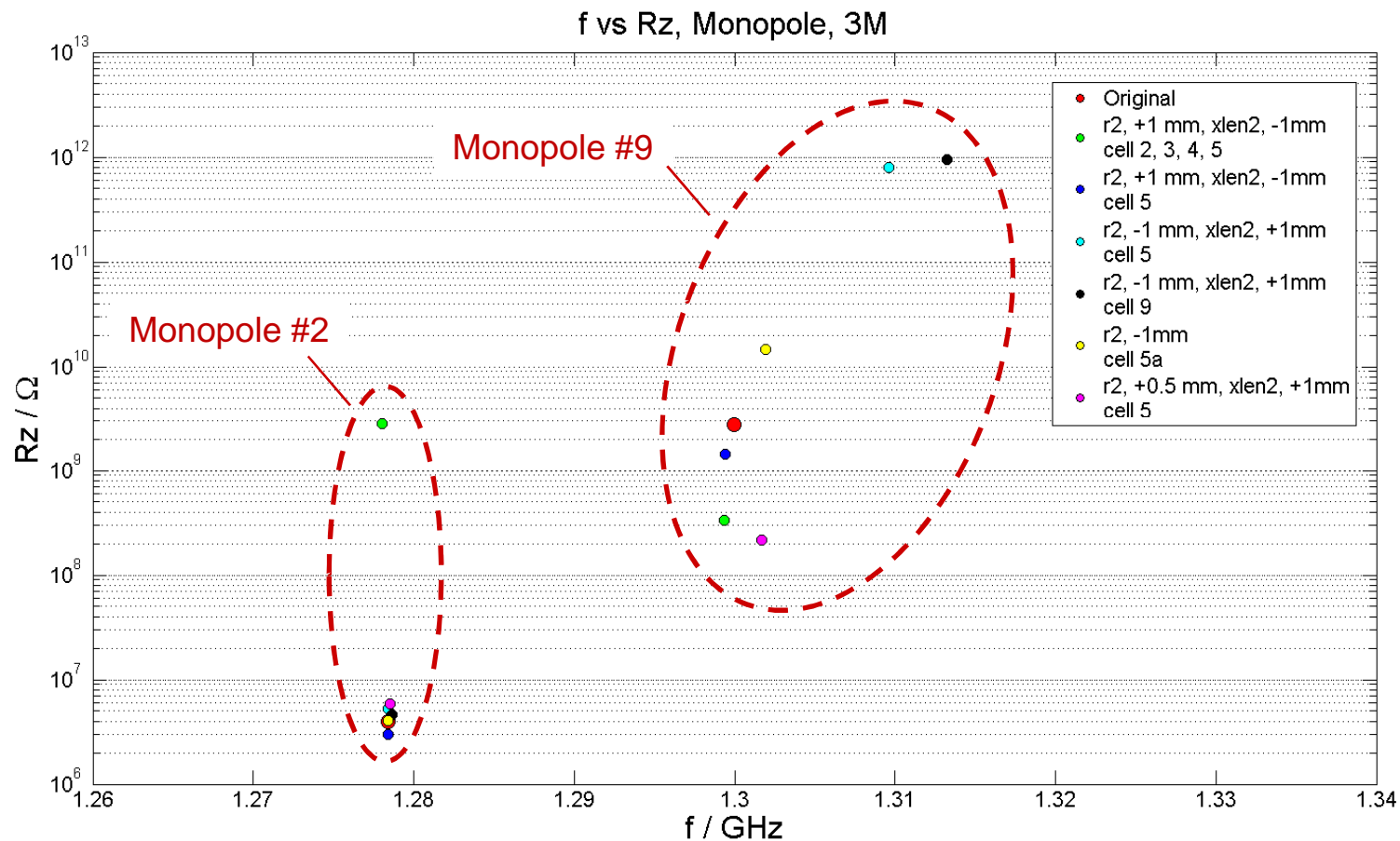
# Numerical Examples

## ▪ Geometry Modifications



# Numerical Examples

## ▪ Geometry Modifications



# Summary / Outlook

## ▪ Summary:

Accurate complex eigenmode solver available

- FEM up to 2<sup>nd</sup> order edge elements
- Geometric modeling with curved tetrahedral elements
- Port boundary conditions with curved triangles
- Application to the 1.3 GHz structure  
(calculation of all modes up the 5<sup>th</sup> dipole passband)

## ▪ Outlook:

- Application to 1.3 GHz and 3.9 GHz cavity strings

